Schnute Growth Model

Schnute, J. 1981. A versatile growth model with statistically stable parameters. Canadian Journal of Fisheries and Aqatic Sciences 38: 1128-1140.

Case 1: $a \neq 0,\ b \neq 0$

(15) $$Y(t) = \left[y_1^{\,b} + (y_2^{\,b} - y_1^{\,b}) \frac{1 - e^{-a(t-\tau_1)}}{1 - e^{-a(\tau_2-\tau_1)}} \right]^{1/b},$$

Case 2: $a \neq 0,\ b = 0$

(16) $$Y(t) = y_1 \exp\left[\log\,(y_2/y_1) \frac{1 - e^{-a(t-\tau_1)}}{1 - e^{-a(\tau_2-\tau_1)}} \right],$$

Case 3: $a = 0,\ b \neq 0$

(17) $$Y(t) = \left[y_1^{\,b} + (y_2^{\,b} - y_1^{\,b}) \frac{t - \tau_1}{\tau_2 - \tau_1} \right]^{1/b},$$

Case 4: $a = 0,\ b = 0$

(18) $$Y(t) = y_1 \exp\left[\log\,(y_2/y_1) \frac{t - \tau_1}{\tau_2 - \tau_1} \right].$$

(24) $$\tau_0 = \begin{cases} \tau_1 + \tau_2 - \dfrac{1}{a} \log\left[\dfrac{e^{a\tau_2} y_2^{\,b} - e^{a\tau_1} y_1^{\,b}}{y_2^{\,b} - y_1^{\,b}} \right]; & a \neq 0,\ b \neq 0; \\ \tau_1 + \tau_2 - \dfrac{\tau_2 y_2^{\,b} - \tau_1 y_1^{\,b}}{y_2^{\,b} - y_1^{\,b}}; & a = 0,\ b \neq 0; \end{cases}$$

(25) $$y_\infty = \begin{cases} \left[\dfrac{e^{a\tau_2} y_2^{\,b} - e^{a\tau_1} y_1^{\,b}}{e^{a\tau_2} - e^{a\tau_1}} \right]^{1/b}; & a \neq 0,\ b \neq 0; \\ \exp\left(\dfrac{e^{a\tau_2} \log y_2 - e^{a\tau_1} \log y_1}{e^{a\tau_2} - e^{a\tau_1}} \right); & a \neq 0,\ b = 0; \end{cases}$$

(26) $$\tau^* = \begin{cases} \tau_1 + \tau_2 - \dfrac{1}{a} \log\left[\dfrac{b(e^{a\tau_2} y_2^{\,b} - e^{a\tau_1} y_1^{\,b})}{y_2^{\,b} - y_1^{\,b}} \right]; & a \neq 0,\ b \neq 0; \\ \tau_1 + \tau_2 - \dfrac{1}{a} \log\left[\dfrac{e^{a\tau_2} - e^{a\tau_1}}{\log\,(y_2/y_1)} \right]; & a \neq 0,\ b = 0; \end{cases}$$

(27) $$y^* = \begin{cases} \left[\dfrac{(1 - b)\,(e^{a\tau_2} y_2^{\,b} - e^{a\tau_1} y_1^{\,b})}{e^{a\tau_2} - e^{a\tau_1}} \right]^{1/b}; & a \neq 0, b \neq 0; \\ \exp\left(\dfrac{(e^{a\tau_2} \log y_2 - e^{a\tau_1} \log y_1)}{e^{a\tau_2} - e^{a\tau_1}} - 1 \right); & a \neq 0,\ b = 0; \end{cases}$$