

AquaEnv - Constants and Formulae

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1 Constants and formulae

1.1 Chemical constants used in AquaEnv

1.1.1 Elements of list PhysChemConst

absZero	-273.15	°C	[Dickson et al., 2007]	absolute zero
R	83.14472	(bar*cm ³)/(mol*K)	[Dickson et al., 2007]	ideal gas constant
F	96485.3399	C/mol	[Dickson et al., 2007]	Faraday constant
e	79	-	[Zeebe and Wolf-Gladrow, 2001]	relative dielectric constant of seawater
K_HNO2	1.584893e-3	mol/l	[Riordan et al., 2005]	approximative dissociation constant of HNO ₂ , NBS pH scale, hybrid constant
K_HNO3	23.44	mol/kg-soln	[Boudreau, 1996, Soetaert et al., 2007]	approximative dissociation constant of HNO ₃ , assumed on mol/kg-soln and free pH scale, stoichiometric constant
K_H2SO4	100	mol/kg-soln	[Atkins, 1996]	approximative dissociation constant of H ₂ SO ₄ , assumed on mol/kg-soln and free pH scale, stoichiometric constant
K_HS	1.1e-12	mol/kg-soln	[Atkins, 1996]	approximative dissociation constant of HS, assumed on mol/kg-soln and free pH scale, stoichiometric constant

1.1.2 Elements of list MeanMolecularMass

The list `MeanMolecularMass` contains mean molecular weights in g/mol. The list is taken from DOE [1994, chap. 5, p. 3] and Dickson et al. [2007, chap. 5, p. 4].

Cl	35.453
SO4	(32.065+4 (15.999))
Br	79.904
F	18.998
Na	22.990
Mg	24.3050
Ca	40.078
K	39.098
Sr	87.62
B	10.811

1.1.3 Elements of list ConcRelCl

The list `ConcRelCl` contains relative concentrations of key chemical species in seawater with respect to chlorinity (DOE [1994, chap. 5, p. 11] and Dickson et al. [2007, chap. 5, p. 10])

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Cl	0.99889
SO4	0.1400
Br	0.003473
F	0.000067
Na	0.55661
Mg	0.06626
Ca	0.02127
K	0.0206
Sr	0.00041
B	0.000232

1.2 Chlorinity Cl as a function of salinity S

Chlorinity Cl (in ‰) is calculated from salinity S using a relation given in DOE [1994, chap. 5, p. 11] and Zeebe and Wolf-Gladrow [2001, p. 100]

$$\text{Cl} = \frac{S}{1.80655} \quad (1)$$

1.3 Total concentrations of key chemical species in seawater as a function of chlorinity Cl

As described in DOE [1994, chap. 5, p. 11] and Dickson et al. [2007, chap. 5, p. 10], values in lists MeanMolecularMass and ConcRelCl\$X are used to calculate the total concentration [X] (in mol/kg-soln) of chemical species X in seawater¹ according to the relation

$$[X] = \frac{\text{ConcRelCl}\$X}{\text{MeanMolecularMass}\$X} \text{Cl} \quad (2)$$

1.4 Ionic strength I as function of salinity S

According to DOE [1994, chapter 5, p. 13, 15], Zeebe and Wolf-Gladrow [2001, p.12], and Roy et al. [1993c, p.257], I (in mol/kg-H₂O) is calculated as

$$I = \frac{19.924 S}{1000 - 1.005 S} \quad (3)$$

Note that the approximation $I/(\text{mol/kg-solution}) \approx 0.0199201 S$ is given in Millero [1982, p. 428.]. This relationship converted into mol/kg-H₂O and the last digits adjusted (from 0.0199201 to 0.019924) results in the formula used here.

1.5 Relation between water depth d and gauge pressure p

Although the relation between gauge pressure p (total pressure minus atmospheric pressure, see Feistel [2008]) and water depth d can be approximated by

$$p = 0.1 d \cdot 1.01325 \quad (4)$$

since p increases per m of water depth d by approximately $\frac{1}{10}$ of 1 atm (= 1.01325 bar Dickson et al. [2007, chap. 5, p. 3]), here, the relation given by Fofonoff and Millard [1983] as implemented in Soetaert et al. [2009] is used

$$d = \frac{(9.72659 + (-2.2512 \cdot 10^{-5} + (2.279 \cdot 10^{-10} - 1.82 \cdot 10^{-15} p) p) p)}{g + 1.092 \cdot 10^{-6} p} \quad (5)$$

¹Note that the solution must have seawater composition, otherwise the relation given here is void.

where p is the gauge pressure in dbar (deci-bar) and g the earth's gravity in m/s^2 . g is calculated from the latitude lat (in degrees, -90 to 90, if not given $lat=0$ is assumed) as given in [Fofonoff and Millard \[1983\]](#) and implemented in [Soetaert et al. \[2009\]](#)

$$g = 9.780318 \left(1 + (0.0052788 + 2.36 \cdot 10^{-5} \sin(lat \frac{\Pi}{180})) \sin(lat \frac{\Pi}{180}) \right) \quad (6)$$

1.6 Seawater density as function of salinity S and temperature t

According to [\[Millero and Poisson, 1981\]](#), [Millero \[1982, p. 434\]](#), as reprinted in [DOE \[1994, chap. 5, p. 6f\]](#) the density of seawater $\rho_{SeaWater}$ (in $\frac{kg}{m^3}$; **density** in an object of class *aquaenv*) can be calculated as

$$\rho_{SeaWater} = \rho_{Water} + A S + B S^{1.5} + C S^2 \quad (7)$$

$$A = 0.824493 - 4.0899 \cdot 10^{-3} t + 7.6438 \cdot 10^{-5} t^2 - 8.2467 \cdot 10^{-7} t^3 \quad (8)$$

$$+ 5.3875 \cdot 10^{-9} t^4 \quad (9)$$

$$B = -5.72466 \cdot 10^{-3} + 1.0227 \cdot 10^{-4} t - 1.6546 \cdot 10^{-6} t^2 \quad (10)$$

$$C = 4.8314 \cdot 10^{-4} \quad (11)$$

$$\rho_{Water} = 999.842594 + 6.793952 \cdot 10^{-2} t - 9.095290 \cdot 10^{-3} t^2 \quad (12)$$

$$+ 1.001685 \cdot 10^{-4} t^3 - 1.120083 \cdot 10^{-6} t^4 + 6.536332 \cdot 10^{-9} t^5 \quad (13)$$

with t representing the temperature in $^{\circ}C$ and ρ_{Water} the density of fresh water in kg/m^3 .

1.7 Gas-exchange constants, dissociation constant, and solubility products as functions of salinity S , (absolute) temperature T , and gauge pressure p

Empirical formulations for the temperature and salinity dependency of all gas exchange constants, equilibrium constants and solubility products calculated in *AquaEnv* can be brought into the generic forms

$$\ln \frac{K_X}{k_0^{\circ}} = A + \frac{B}{T} + C \ln(T) + D T + E T^2 \quad (14)$$

or

$$\log_{10} \frac{K_X}{k_0^{\circ}} = A' + \frac{B'}{T} + C' \log_{10}(T) + D' T + E' T^2 \quad (15)$$

or

$$\log_{10} \frac{K_X}{k_0^{\circ}} = A'' + \frac{B''}{T} + C'' \ln(T) + D'' T + E'' T^2 \quad (16)$$

with T being the temperature in Kelvin, S the salinity, k_0° the concentration unit of the constant, and A, B, C, D, E , and the respective variables with a prime ($'$) being functions of salinity S . In the following we will give A, B, C, D , and E , or A', B', C', D' , and E' , or A'', B'', C'', D'' , and E'' for each calculated constant.

1.7.1 Gas-exchange constants (Henry's constants) as functions of salinity S and temperature T

The following table shows the coefficients for gas exchange constants in *AquaEnv*, with fCO_2 being the fugacity of CO_2 .

K0_CO2 : solubility of CO ₂ in seawater	
A = 0.023517S - 167.81077 B = 9345.17 C = 23.3585 D = -2.3656 10 ⁻⁴ S E = 4.7036 10 ⁻⁷ S	CO2_sat = fCO ₂ K0_CO2 $k_0^\circ = \left[\frac{\text{mol}}{\text{kg-solution atm}} \right]$
References: Weiss [1974] (original), DOE [1994, chap. 5, p. 13], Millero [1995, p. 663], Zeebe and Wolf-Gladrow [2001, p. 257], and Dickson et al. [2007, chap. 5, p. 12]	
K0_O2 : solubility of O ₂ in seawater (micromol per kg-soln and atm)	
A = -846.9978 - 0.037362 S B = 25559.07 C = 146.4813 D = -0.22204 + 0.00016504 S E = -2.0564 10 ⁻⁷ S	O2_sat = fO ₂ K0_O2 $k_0^\circ = \left[\frac{\mu\text{mol}}{\text{kg-solution atm}} \right]$
References: derived from Weiss [1970], agrees with data in Murray and Riley [1969]	

Note that the formulation for K0_O2 has been derived using the formulation for a gravimetric [O₂]_{sat} given in Weiss [1970, Weiss, 1970]. It has been converted from ml-O₂/kg-soln to μmol-O₂/kg-soln using the molar volume of O₂ calculated with the virial equation using a first virial coefficient for oxygen at 273.15 Kelvin of -22 cm³/mol Atkins [1996], a value of 8.314472 Nm/(Kelvin mol) for the gas constant R and an ambient pressure of 101325 N/m². The expression for the Henry's constant has then been created by dividing the expression for the saturation concentration by fO₂ = 0.20946 atm [Williams, 2004].

1.7.2 Stoichiometric acid base dissociation constants as functions of salinity S and temperature T

The following table gives the coefficients of stoichiometric acid base dissociation constants in AquaEnv. Note that if some of the coefficients A to E are not listed, they are to be considered zero. Note also that given references sometimes contain the formulae in different units or on different pH scales. The formulae provided in this table yield the dissociation constants on different pH scales and concentration units. In AquaEnv, constants that are not already on the free pH scale and in mol/kg-soln are converted to the free pH scale and mol/kg-soln.

K_HS04 : HSO ₄ ⁻ ⇌ H ⁺ + SO ₄ ²⁻ ("dickson")		free pH scale
A = 324.57 $\sqrt{\left(\frac{I}{m^\circ}\right)} - 771.54 \frac{I}{m^\circ} + 141.328$ B = 35474 $\frac{I}{m^\circ} + 1776 \left(\frac{I}{m^\circ}\right)^2 - 13856 \sqrt{\left(\frac{I}{m^\circ}\right)} - 2698 \left(\frac{I}{m^\circ}\right)^{\frac{3}{2}} - 4276.1$ C = 114.723 $\frac{I}{m^\circ} - 47.986 \sqrt{\left(\frac{I}{m^\circ}\right)} - 23.093$	K_HS04 = $k^\circ =$ $m^\circ =$	$\frac{[H^+]_F [SO_4^{2-}]}{[HSO_4^-]}$ $\frac{\text{mol}}{\text{kg-H}_2\text{O}}$ $\frac{\text{mol}}{\text{kg-H}_2\text{O}}$
References: DOE [1994, c. 5, p. 13], Zeebe and Wolf-Gladrow [2001, p. 260], Dickson [1990b] (original)		
K_HS04 : HSO ₄ ⁻ ⇌ H ⁺ + SO ₄ ²⁻ ("khoo")		free pH scale
A = 6.3451 + 0.5208 $\sqrt{\left(\frac{I}{m^\circ}\right)}$ B = -647.59 D = -0.019085	K_HS04 = $k^\circ =$ $m^\circ =$	$\frac{[H^+]_F [SO_4^{2-}]}{[HSO_4^-]}$ $\frac{\text{mol}}{\text{kg-H}_2\text{O}}$ $\frac{\text{mol}}{\text{kg-H}_2\text{O}}$
References: Khoo et al. [1977] (original), Roy et al. [1993b], Millero [1995], Lewis and Wallace [1998]		

K_HF: $\text{HF} \rightleftharpoons \text{H}^+ + \text{F}^-$ ("dickson")		free pH scale
A = $1.525 \sqrt{\frac{I}{m^\circ}} - 12.641$ B = 1590.2	K_HF = $\frac{[\text{H}^+]_F [\text{F}^-]}{[\text{HF}]}$ $k^\circ = m^\circ = \frac{\text{mol}}{\text{kg-H}_2\text{O}}$	
References: Dickson and Riley [1979a, p. 91] (original), Dickson and Millero [1987, p. 1783], Roy et al. [1993b, p. 257], DOE [1994, c. 5, p. 15], Millero [1995, p. 664], Zeebe and Wolf-Gladrow [2001, p. 260]		
K_HF: $\text{HF} \rightleftharpoons \text{H}^+ + \text{F}^-$ ("perez")		total pH scale
A = $-9.68 + 0.111 \sqrt{S}$ B = 874	K_HF = $\frac{[\text{H}^+]_F [\text{F}^-]}{[\text{HF}]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$	
References: Perez and Fraga [1987, p. 91] (original), Dickson et al. [2007, chap. 5, p. 14]		
K_C02: $\text{CO}_2(\text{aq}) + \text{H}_2\text{O} (\rightleftharpoons \text{H}_2\text{CO}_3) \rightleftharpoons \text{H}^+ + \text{HCO}_3^-$ ("roy"; high salinities: $S > 5$)		total pH scale
A = $2.83655 - 0.20760841 \sqrt{S} + 0.08468345 S - 0.00654208 S^{\frac{3}{2}}$ B = $-2307.1266 - 4.0484 \sqrt{S}$ C = -1.5529413	K_C02 = $\frac{[\text{H}^+] [\text{HCO}_3^-]}{[\text{CO}_2(\text{aq})]}$ $k^\circ = \frac{\text{mol}}{\text{kg-H}_2\text{O}}$	
References: Roy et al. [1993b, p. 254] (original), DOE [1994, c. 5, p.14], Millero [1995, p. 664], Zeebe and Wolf-Gladrow [2001, p. 255]		
K_C02: $\text{CO}_2(\text{aq}) + \text{H}_2\text{O} (\rightleftharpoons \text{H}_2\text{CO}_3) \rightleftharpoons \text{H}^+ + \text{HCO}_3^-$ ("roy"; low salinities: $S \leq 5$)		total pH scale
A = $290.9097 - 228.39774 \sqrt{S} + 54.20871 S - 3.969101 S^{\frac{3}{2}} - 0.00258768 S^2$ B = $-14554.21 + 9714.36839 \sqrt{S} - 2310.48919 S + 170.22169 S^{\frac{3}{2}}$ C = $-45.0575 + 34.485796 \sqrt{S} - 8.19515 S + 0.60367 S^{\frac{3}{2}}$	K_C02 = $\frac{[\text{H}^+] [\text{HCO}_3^-]}{[\text{CO}_2(\text{aq})]}$ $k^\circ = \frac{\text{mol}}{\text{kg-H}_2\text{O}}$	
References: Roy et al. [1993b, p. 256] (original, based on a temperature dependency restated in Millero [1979], originally given in Harned and Davis [1943]. Note that there is a typesetting error in Roy et al. [1993b]: The third value for B is 2310.48919, not 310.48919) Millero [1995, p. 664] (the typesetting error is corrected here. also, here it is mentioned that this formula should be used for $S \leq 5$. Note that both functions do not always intersect at $S=5$. The true intersection is a function of t , is calculated in AquaEnv, and is used to decide which formula to use.)		
K_C02: $\text{CO}_2(\text{aq}) + \text{H}_2\text{O} (\rightleftharpoons \text{H}_2\text{CO}_3) \rightleftharpoons \text{H}^+ + \text{HCO}_3^-$ ("lueker")		total pH scale
A" = $61.2172 + 0.011555 S - 0.0001152 S^2$ B" = -3633.86 C" = -9.67770	K_C02 = $\frac{[\text{H}^+] [\text{HCO}_3^-]}{[\text{CO}_2(\text{aq})]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$	
References: Lueker et al. [2000] (original), Dickson et al. [2007, chap. 5, p.13-14]		
K_C02: $\text{CO}_2(\text{aq}) + \text{H}_2\text{O} (\rightleftharpoons \text{H}_2\text{CO}_3) \rightleftharpoons \text{H}^+ + \text{HCO}_3^-$ ("millero")		SW pH scale
A" = $126.34048 - 0.0331 S + 0.0000533 S^2 - 13.4191 \sqrt{S}$ B" = $-6320.813 + 6.103 S + 530.123 \sqrt{S}$ C" = $-19.568224 + 2.06950 \sqrt{S}$	K_C02 = $\frac{[\text{H}^+] [\text{HCO}_3^-]}{[\text{CO}_2(\text{aq})]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$	
References: Millero et al. [2006] (original)		
K_HC03: $\text{HCO}_3^- \rightleftharpoons \text{H}^+ + \text{CO}_3^{2-}$ ("roy"; high salinities: $S > 5$)		total pH scale
A = $-9.226508 - 0.106901773 \sqrt{S} + 0.1130822 S - 0.00846934 S^{\frac{3}{2}}$ B = $-3351.6106 - 23.9722 \sqrt{S}$ C = -0.2005743	K_HC03 = $\frac{[\text{H}^+] [\text{CO}_3^{2-}]}{[\text{HCO}_3^-]}$ $k^\circ = \frac{\text{mol}}{\text{kg-H}_2\text{O}}$	
References: Roy et al. [1993b, p. 254] (original), DOE [1994, c. 25, p.15], Millero [1995, p. 664], Zeebe and Wolf-Gladrow [2001, p. 255]		

K_HC03: $\text{HCO}_3^- \rightleftharpoons \text{H}^+ + \text{CO}_3^{2-}$ ("roy"; low salinities: $S \leq 5$)		total pH scale
$A = 207.6548 - 167.69908 \sqrt{S} + 39.75854 S - 2.892532 S^{\frac{3}{2}} - 0.00613142 S^2$ $B = -11843.79 + 6551.35253 \sqrt{S} - 1566.13883 S + 116.270079 S^{\frac{3}{2}}$ $C = -33.6485 + 25.928788 \sqrt{S} - 6.171951 S + 0.45788501 S^{\frac{3}{2}}$		$K_{\text{HC03}} = \frac{[\text{H}^+][\text{CO}_3^{2-}]}{[\text{HCO}_3^-]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$
<i>References:</i> Roy et al. [1993b, p. 256] (original, based on a temperature dependence restated in Millero [1979], originally given in Harned and Scholes [1941]), Millero [1995, p. 664] (here it is mentioned that this formula should be used for $S \leq 5$. Note that both functions do not always intersect at $S=5$. The true intersection is a function of t , is calculated in AquaEnv, and is used to decide which formula to use.)		
K_HC03: $\text{HCO}_3^- \rightleftharpoons \text{H}^+ + \text{CO}_3^{2-}$ ("lueker")		total pH scale
$A'' = -25.9290 + 0.01781 S - 0.0001122 S^2$ $B'' = -471.78$ $C'' = 3.16967$		$K_{\text{HC03}} = \frac{[\text{H}^+][\text{CO}_3^{2-}]}{[\text{HCO}_3^-]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$
<i>References:</i> Lueker et al. [2000] (original), Dickson et al. [2007, chap. 5, p.14]		
K_HC03: $\text{HCO}_3^- \rightleftharpoons \text{H}^+ + \text{CO}_3^{2-}$ ("millero")		SW pH scale
$A'' = 90.18333 - 0.1248 S + 0.0003687 S^2 - 21.0894 \sqrt{S}$ $B'' = -5143.692 + 20.051 S + 772.483 \sqrt{S}$ $C'' = -14.613358 + 3.3336 \sqrt{S}$		$K_{\text{HC03}} = \frac{[\text{H}^+][\text{CO}_3^{2-}]}{[\text{HCO}_3^-]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$
<i>References:</i> Millero et al. [2006] (original)		
K_W: $\text{H}_2\text{O} \rightleftharpoons \text{H}^+ + \text{OH}^-$		total pH scale
$A = 148.9652 - 5.977 \sqrt{S} - 0.01615 S$ $B = -13847.26 + 118.67 \sqrt{S}$ $C = -23.6521 + 1.0495 \sqrt{S}$		$K_W = \frac{[\text{H}^+][\text{OH}^-]}{1}$ $k^\circ = \left(\frac{\text{mol}}{\text{kg-solution}} \right)^2$
<i>References:</i> Millero [1995, p.670] (original), DOE [1994, c. 5, p. 18] (update 1997 cites Millero [1995]), Zeebe and Wolf-Gladrow [2001, p. 258], Dickson et al. [2007, chap. 5, p.16]		
K_BOH3: $\text{B(OH)}_3 \rightleftharpoons \text{H}^+ + \text{B(OH)}_4^-$		total pH scale
$A = 148.0248 + 137.1942 \sqrt{S} + 1.62142S$ $B = -8966.90 - 2890.53 \sqrt{S} - 77.942S + 1.728 S^{\frac{3}{2}} - 0.0996 S^2$ $C = -24.4344 - 25.085 \sqrt{S} - 0.2474 S$ $D = 0.053105 \sqrt{S}$		$K_{\text{BOH3}} = \frac{[\text{H}^+][\text{B(OH)}_4^-]}{[\text{B(OH)}_3]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$
<i>References:</i> Dickson [1990a, p. 763] (or.), DOE [1994, c. 5, p. 14], Millero [1995, p. 669], Zeebe and Wolf-Gladrow [2001, p. 262], agrees with data in Roy et al. [1993a]		
K_NH4: $\text{NH}_4^+ \rightleftharpoons \text{H}^+ + \text{NH}_3$		SW pH scale
$A = -0.25444 + 0.46532 \sqrt{S} - 0.01992 S$ $B = -6285.33 - 123.7184 \sqrt{S} + 3.17556 S$ $D = 0.0001635$		$K_{\text{NH4}} = \frac{[\text{H}^+][\text{NH}_3]}{[\text{NH}_4^+]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$
<i>References:</i> Millero [1995, p. 671], Millero et al. [1995] (original), corrections of Millero [1995] in Lewis and Wallace [1998] give pH scale		
K_H2S: $\text{H}_2\text{S} \rightleftharpoons \text{H}^+ + \text{HS}^-$		total pH scale
$A = 225.838 + 0.3449 \sqrt{S} - 0.0274 S$ $B = -13275.3$ $C = -34.6435$		$K_{\text{H2S}} = \frac{[\text{H}^+][\text{HS}^-]}{[\text{H}_2\text{S}]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$
<i>References:</i> Millero [1995, p. 671], Millero et al. [1988] (original), corrections of Millero [1995] in Lewis and Wallace [1998] give pH scale		

K_H3P04: $\text{H}_3\text{PO}_4 \rightleftharpoons \text{H}^+ + \text{H}_2\text{PO}_4^-$		total pH scale
$A = 115.525 + 0.69171 \sqrt{S} - 0.01844 S$ $B = -4576.752 - 106.736 \sqrt{S} - 0.65643 S$ $C = -18.453$	$K_{\text{H3P04}} = \frac{[\text{H}^+][\text{H}_2\text{PO}_4^-]}{[\text{H}_3\text{PO}_4]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$	
<i>References:</i> DOE [1994, chap. 5, p 16], Millero [1995, p.670], (original) Dickson et al. [2007, chap. 5, p.15] agrees with data in Dickson and Riley [1979b]		
K_H2P04 : $\text{H}_2\text{PO}_4^- \rightleftharpoons \text{H}^+ + \text{HPO}_4^{2-}$		total pH scale
$A = 172.0883 + 1.3566 \sqrt{S} - 0.05778 S$ $B = -8814.715 - 160.340 \sqrt{S} + 0.37335 S$ $C = -27.927$	$K_{\text{H2P04}} = \frac{[\text{H}^+][\text{HPO}_4^{2-}]}{[\text{H}_2\text{PO}_4^-]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$	
<i>References:</i> DOE [1994, chap. 5, p 16], Millero [1995, p.670] (original), Dickson et al. [2007, chap. 5, p.15], agrees with data in Dickson and Riley [1979b]		
K_HP04 : $\text{HPO}_4^{2-} \rightleftharpoons \text{H}^+ + \text{PO}_4^{3-}$		total pH scale
$A = -18.141 + 2.81197 \sqrt{S} - 0.09984 S$ $B = -3070.75 + 17.27039 \sqrt{S} - 44.99486 S$	$K_{\text{HP04}} = \frac{[\text{H}^+][\text{PO}_4^{3-}]}{[\text{HPO}_4^{2-}]}$ $k^\circ = \frac{\text{mol}}{\text{kg-solution}}$	
<i>References:</i> DOE [1994, chap. 5, p 17], Millero [1995, p.670] (original), Dickson et al. [2007, chap. 5, p.15], agrees with data in Dickson and Riley [1979b]		
K_Si0H4: $\text{Si}(\text{OH})_4 \rightleftharpoons \text{H}^+ + \text{SiO}(\text{OH})_3^-$		total pH scale
$A = 117.385 + 3.5913 \sqrt{\frac{I}{m^\circ}} - 1.5998 \frac{I}{m^\circ} + 0.07871 \left(\frac{I}{m^\circ}\right)^2$ $B = -8904.2 - 458.79 \sqrt{\frac{I}{m^\circ}} + 188.74 \frac{I}{m^\circ} - 12.1652 \left(\frac{I}{m^\circ}\right)^2$ $C = -19.334$	$K_{\text{Si0H4}} = \frac{[\text{H}^+][\text{SiO}(\text{OH})_3^-]}{[\text{Si}(\text{OH})_4]}$ $k^\circ = \frac{\text{mol}}{\text{kg-H}_2\text{O}}$ $m^\circ = \frac{\text{mol}}{\text{kg-H}_2\text{O}}$	
<i>References:</i> Millero et al. [1988] (original), DOE [1994, chapter 5, p 17], Millero [1995, p.671]		
K_Si00H3: $\text{SiO}(\text{OH})_3^- \rightleftharpoons \text{H}^+ + \text{SiO}_2(\text{OH})_2^{2-}$		total pH scale
$A = 8.96$ $B = -4465.18$ $D = 0.021952$	$K_{\text{Si00H3}} = \frac{[\text{H}^+][\text{SiO}_2(\text{OH})_2^{2-}]}{[\text{SiO}(\text{OH})_3^-]}$ $k^\circ = \frac{\text{mol}}{\text{kg-H}_2\text{O}}$	
<i>References:</i> Wischmeyer et al. [2003] (original; including corrections by co-author D. Wolf-Gladrow)		

1.8 Stoichiometric solubility products as functions of salinity S and temperature T

The following table shows the coefficients for the stoichiometric solubility products for calcite and aragonite in AquaEnv.

Ksp_calcite : solubility product of calcite	
$A' = -171.9065 - 0.77712 \sqrt{S} - 0.07711 S + 0.0041249 S^{1.5}$ $B' = 2839.319 + 178.34 \sqrt{S}$ $C' = 71.595$ $D' = -0.077993 + 0.0028426 \sqrt{S}$	$K_{\text{sp-cal}} = [\text{CO}_3^{2-}][\text{Ca}^{2+}]$ $k_0^\circ = \left[\left(\frac{\text{mol}}{\text{kg-solution}} \right)^2 \right]$
<i>References:</i> Mucci [1983] (original), Boudreau [1996, p. 160], (note that the second value for A' is -0.77712 not -0.7712 as cited in Boudreau [1996])	

Ksp_aragonite : solubility product of aragonite	
$A' = -171.945 - 0.068393 \sqrt{S} - 0.10018 S + 0.0059415 S^{1.5}$ $B' = 2903.293 + 88.135 \sqrt{S}$ $C' = 71.595$ $D' = -0.077993 + 0.0017276 \sqrt{S}$	$K_{sp_ara} = [CO_3^{2-}] [Ca^{2+}]$ $k_0^\circ = \left[\left(\frac{mol}{kg-solution} \right)^2 \right]$
<i>References:</i> Mucci [1983] (original), Boudreau [1996, p. 160] , (note that the second value for D' is 0.0017276 not 0.001727 as cited in Boudreau [1996])	

1.9 Pressure correction of dissociation constants and solubility products

Pressure has an effect on the stoichiometric acid-base dissociation constants and the stoichiometric solubility products given in the previous sections. As described in [Millero \[1995, p. 675\]](#) using corrections and assumptions from [Lewis and Wallace \[1998, p. A-7\]](#) the effect of pressure can be accounted for by the equation

$$K_{corr} = K \left(-\frac{a_0 + a_1 t + a_2 t^2}{R T} p + \frac{b_0 + b_1 t + b_2 t^2}{1000 R T} 0.5 p^2 \right) \quad (17)$$

Where K_{corr} is the pressure corrected constant and K is the uncorrected constant, both on matching units, e.g., mol/kg-soln, T is the absolute temperature in Kelvin, t is the temperature in °C, R is the ideal gas constant in (bar cm³)/(mol Kelvin), and p is the gauge pressure (total pressure minus one atm, see [Feistel \[2008\]](#) for a definition) in bar. The a and b coefficients (according to [Millero \[1995\]](#) which is partly a restatement of [Millero \[1979\]](#), corrected by [Lewis and Wallace \[1998\]](#)) for constants in AquaEnv (stored in the data frame `DeltaPcoeffs`) are given in the following table².

Note that, while not stated in [Millero \[1995\]](#), it can be inferred from [Lewis and Wallace \[1998\]](#) and the code given by [van Heuven et al. \[2009\]](#), that the pressure correction is valid for `K_HF` and `H_H2SO4` on the free scale and for all other dissociation constants on the seawater pH scale. To be consistent with [Lewis and Wallace \[1998\]](#) and [van Heuven et al. \[2009\]](#), in AquaEnv all dissociation constants obtained from the original formulae are first converted to the free or seawater scale respectively using scale conversion factors with `K_HF` and `H_H2SO4` being not pressure corrected. Then the pressure correction is applied. Subsequently, the dissociation constants are converted to the desired pH scale with scale conversion factors with `K_HF` and `H_H2SO4` being pressure corrected.

	a_0	a_1	a_2	b_0	b_1	b_2
K_HS04	-18.03	0.0466	0.3160 10 ⁻³	- 4.53	0.0900	0
K_HF	-9.78	-0.0090	-0.9420 10 ⁻³	- 3.91	0.0540	0
K_CO2	-25.50	0.1271	0.0000 10 ⁻³	- 3.08	0.0877	0
K_HCO3	-15.82	-0.0219	0.0000 10 ⁻³	1.13	-0.1475	0
K_W	-25.60	0.2324	-3.6246 10 ⁻³	- 5.13	0.0794	0
K_BOH3	-29.48	0.1622	2.6080 10 ⁻³	- 2.84	0.0000	0
K_NH4	-26.43	0.0889	-0.9050 10 ⁻³	- 5.03	0.0814	0
K_H2S	-14.80	0.0020	-0.4000 10 ⁻³	2.89	0.0540	0
K_H3PO4	-14.51	0.1211	-0.3210 10 ⁻³	- 2.67	0.0427	0
K_H2PO4	-23.12	0.1758	-2.6470 10 ⁻³	- 5.15	0.0900	0
K_HPO4	-26.57	0.2020	-3.0420 10 ⁻³	- 4.08	0.0714	0
K_SiOH4	-29.48	0.1622	2.6080 10 ⁻³	- 2.84	0.0000	0
K_SiOOH3	-29.48	0.1622	2.6080 10 ⁻³	- 2.84	0.0000	0
Ksp_calcite	-48.76	0.5304	0.0000 10 ⁻³	-11.76	0.3692	0
Ksp_aragonite	-45.96	0.5304	0.0000 10 ⁻³	-11.76	0.3692	0

²Note that in [Lewis and Wallace \[1998\]](#) it is stated that the a values for H₂O and H₂S are *freshwater* values! And that the coefficients for the silicate species are assumed to be the same as the ones for the borate species.

1.10 Conversion factors

The following list gives a basic list of concentration and pH scale conversion factors used in AquaEnv. All other conversion factors, e.g., to be used in the function `convert`, are calculated from the factors given here. Note that the factors given below are multiplicative factors that can be used to convert e.g. dissociation constants or proton concentration values. To convert pH values, one needs to use the negative decadal logarithm of the factors below as an additive term. `molal2molin` signifies conversion from mol/kg-H₂O to mol/kg-soln, `free2tot` signifies conversion from the free to the total pH scale, `free2sws` signifies conversion from the free to the seawater pH scale (for a general treatment of the free, total and seawater pH scale see [Dickson \[1984\]](#) and [Zeebe and Wolf-Gladrow \[2001\]](#)), and `free2nbs` signifies conversion from the free to the NBS pH scale [\[Durst, 1975\]](#).

<code>molal2molin</code>	$(1 - 0.001005 S)$	Roy et al. [1993b, p. 257] , DOE [1994, chap. 5, p. 15]
<code>free2tot</code>	$(1 + \frac{S_T}{K_{\text{HSO4}}})$	Dickson [1984, p. 2302] , DOE [1994, chap. 5, p. 16] , Zeebe and Wolf-Gladrow [2001, p. 57, p. 261]
<code>free2sws</code>	$(1 + \frac{S_T}{K_{\text{HSO4}}} + \frac{F_T}{K_{\text{HF}}})$	Dickson [1984, p. 2303] , Zeebe and Wolf-Gladrow [2001]
<code>free2nbs</code>	$\frac{\gamma_{\text{H}^+} \text{free2sws}}{\text{molal2molin}}$	Dickson [1984] , Lewis and Wallace [1998] , Zeebe and Wolf-Gladrow [2001]

In the above table S is salinity, $S_T = [\text{SO}_4^{2-}] + [\text{HSO}_4^-] \approx [\text{SO}_4^{2-}]$, $F_T = [\text{HF}] + [\text{F}^-] \approx [\text{F}^-]$, both in mol/kg-soln, and γ_{H^+} is the activity coefficient for the proton. The dissociation constants K_{HSO4} and K_{HF} are on the free pH scale and in mol/kg-soln as well. Note that, as given in [Dickson \[1984, p. 2303\]](#) and [Dickson and Riley \[1979a, p. 91f\]](#) all concentrations appearing in the definition for the total and the seawater pH scale are molal, i.e. mol/kg-H₂O, concentrations. But in [Roy et al. \[1993b, p. 257\]](#) and in [DOE \[1994, chap. 4, SOP 6, p. 1\]](#) it is stated, that concentrations for the seawater and total pH scale are molin, i.e. mol/kg-soln. To be consistent with [DOE \[1994\]](#) and [Dickson et al. \[2007\]](#) mol/kg-soln is chosen here for the free, total and seawater scale. To be consistent with [Lewis and Wallace \[1998\]](#), the NBS scale is based on the proton concentration on mol/kg-H₂O.

1.11 Activity coefficient for the proton

In AquaEnv a complex ion-interaction model like, e.g., [Millero and Pierrot \[1998\]](#) is not implemented. According to [Zeebe and Wolf-Gladrow \[2001\]](#) the activity coefficient for the proton γ_{H^+} can be approximated by the Davies equation as long as the ionic strength of the solution in question remains below 0.5 mol/kg-H₂O. This means for solutions with a salinity of less than 24.48. Since NBS scale pH values are mostly not used for open ocean applications but mainly in brackish and fresh waters, the Davies equation has been assumed to be a sufficient approximation for γ_{H^+} . Important to note, however, is that **the conversion from and to the NBS pH scale in AquaEnv for salinities above 24.48 is only a rough approximation!**. The Davies equation is used as given in [Zeebe and Wolf-Gladrow \[2001\]](#)

$$\gamma_{\text{H}^+} = 10^{-\left(1.82 \cdot 10^6 (\epsilon T)^{-\frac{3}{2}}\right) \left(\frac{\sqrt{I}}{1+\sqrt{I}} - 0.2 I\right)} \quad (18)$$

where ϵ is the relative dielectric constant of seawater (`PhysChemConst$e` in AquaEnv), T is the temperature in Kelvin, and I is the ionic strength in mol/kg-H₂O. Note that the squared charge of the ion before the brackets with the ionic strength terms which is present in the generic form of the Davies equation has been omitted here since for the proton, this factor is 1.

1.12 The revelle factor

In Zeebe and Wolf-Gladrow [2001, p.73] the revelle factor is given as

$$\text{revelle} = \frac{d[\text{CO}_2]}{[\text{CO}_2]} \bigg/ \frac{d[\sum \text{CO}_2]}{[\sum \text{CO}_2]} \bigg|_{[\text{TA}]=\text{const.}} \quad (19)$$

in AquaEnv `revelle` is calculated numerically.

1.13 Partial derivatives of total alkalinity

The values for `dTAdKdKdS`, `dTAdKdKdT`, `dTAdKdKdd`, `dTAdKdKdSumH2SO4`, and `dTAdKdKdSumHF` are calculated numerically as described in ?.

The values for `dTAdH`, `dTAdSumCO2`, `dTAdSumBOH3`, `dTAdSumH2SO4`, and `dTAdSumHF` are calculated analytically as given in Hofmann et al. [2008].

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