

Solving Differential Equations in R (book) - DAE examples

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Abstract

This vignette contains the R-examples of chapter 6 from the book:
Soetaert, K., Cash, J.R. and Mazzia, F. (2012). Solving Differential Equations in R.
that will be published by Springer.

Chapter 6. Solving Differential Algebraic Equations in R.

Here the code is given without documentation. Of course, much more information
about each problem can be found in the book.

Keywords: differential algebraic equations, initial value problems, examples, R.

1. A simple DAE of Index 2

```
resdae <- function (t, y, dy, p) {  
  r1 <- dy[1] - y[2]  
  r2 <- y[1] - cos(t)  
  list(c(r1, r2))  
}  
library(deTestSet)  
yini <- c(y1 = cos(0), y2 = -sin(0))  
dyini <- c(-sin(0), -cos(0))  
times <- seq(from = 0, to = 10, by = 0.1)  
index <- c(1, 1, 0)  
out1 <- mebdfi(times = times, res = resdae, y = yini,  
               atol = 1e-10, rtol = 1e-10, dy = dyini,  
               parms = NULL, nind = index)  
max (abs(out1[, "y1"] - cos(times)), abs(out1[, "y2"] + sin(times)))
```

```
[1] 2.349123e-09
```

```
fundae <- function (t, y, p) {  
  f1 <- y[2]  
  f2 <- y[1] - cos(t)  
  list(c(f1, f2))  
}
```

```
}  
M <- matrix(nrow = 2, ncol = 2, data = c(1, 0, 0, 0))  
out2 <- radau(times = times, fun = fundae, y = yini,  
              atol = 1e-10, rtol = 1e-10, mass = M,  
              parms = NULL, nind = index)  
max (abs(out2[, "y1"] - cos(times)), abs(out2[, "y2"] + sin(times)))  
  
[1] 5.476366e-07
```

2. A Nonlinear Implicit DAE of index 1

```

implicit <- function(t, y, dy, parms) {
  list(t*y^2*dy^3 - y^3*dy^2 + t*(t^2+1)*dy - t^2*y)
}
yini <- sqrt(3/2)
times <- seq(from = 1, to = 10, by = 0.1)
library(rootSolve)
rootfun <- function (dy, y, t)
  t*y^2*dy^3 - y^3*dy^2 + t*(t^2+1)*dy - t^2*y
dyini <- multiroot(f = rootfun, start = 0, y = yini,
  t = times[1] )$root
dyini

[1] 0.8164966

out <- mebdfi(times = times, res = implicit, y = yini,
  dy = dyini, parms = NULL)
out2 <- daspk (times = times, res = implicit, y = yini,
  dy = dyini, parms = NULL)
max(abs(out [,2]- sqrt(times^2+0.5)))

[1] 3.017694e-06

max(abs(out2[,2]- sqrt(times^2+0.5)))

[1] 5.689474e-05

implicit2 <- function (t, y, p) {
  f1 <- y[2]
  f2 <- t*y[1]^2*y[2]^3-y[1]^3*y[2]^2+t*(t^2+1)*y[2]-t^2*y[1]
  list(c(f1, f2))
}
M <- matrix(nrow = 2, ncol = 2, data = c(1, 0, 0, 0))
yini_li <- c(yini,dyini)
out3 <- radau(times = times, fun = implicit2, y = yini_li,
  mass = M, parms = NULL)
out4 <- gamd (times = times, fun = implicit2, y = yini_li,
  mass = M, parms = NULL)
max(abs(out3[,2]- sqrt(times^2+0.5)))

[1] 3.41116e-08

max(abs(out4[,2]- sqrt(times^2+0.5)))

[1] 1.990122e-06

```

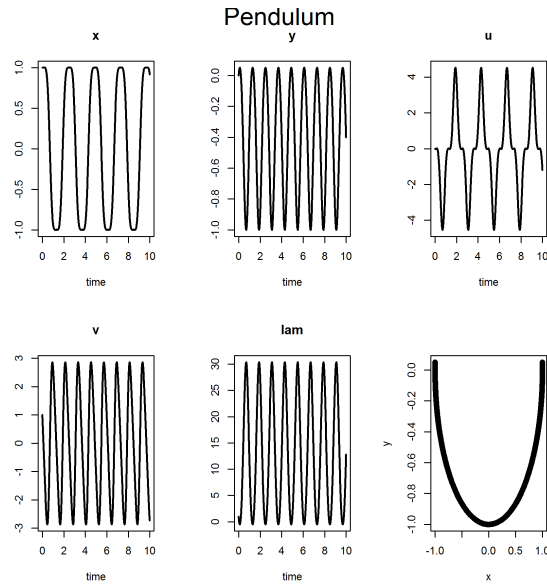


Figure 1: Solution of the pendulum problem. See book for explanation.

3. The Pendulum Problem

```
library(deTestSet)
pendulum <- function (t, y, dy, parms) {
  list(c(-dy[1] + y[3]          ,
        -dy[2] + y[4]          ,
        -dy[3] -y[5]*y[1]      ,
        -dy[4] -y[5]*y[2] - 9.8,
        y[1]^2 + y[2]^2 -1
      ))
}

yini  <- c(x = 1,  y = 0, u = 0,  v = 1  , lam = 1)
dyini <- c(dx = 0, dy = 1, du = -1, dv = -9.8, dlam = 3*9.8)
times <- seq(from = 0, to = 10, by = 0.01)
index3 <- c(2, 2, 1)
out3 <- mebdfi (y = yini, dy = dyini, res = pendulum,
               parms = NULL, times = times,
               nind = index3)

plot(out3, lwd = 2)
plot(out3[, 2:3])
mtext(side = 3, outer = TRUE, line = -1.5,
      "Pendulum", cex = 1.5)
```

4. The Car Axis problem

```

caraxis <- function(t, y, dy, parms) {
  with(as.list(y), {
    f <- rep(0, 10)
    yb <- r * sin(w * t)
    xb <- sqrt(L^2 - yb^2)
    L1 <- sqrt(xl^2 + yl^2)
    Lr <- sqrt((xr - xb)^2 + (yr - yb)^2)
    f[1:4] <- y[5:8]
    f[5] <- 1/k*((L0-L1)*xl/L1 + lam1*xb + 2*lam2*(xl-xr))
    f[6] <- 1/k*((L0-L1)*yl/L1 + lam1*yb + 2*lam2*(yl-yr)) -g
    f[7] <- 1/k*((L0-Lr)*(xr - xb)/Lr - 2*lam2*(xl-xr))
    f[8] <- 1/k*((L0-Lr)*(yr - yb)/Lr - 2*lam2*(yl-yr)) -g
    f[9] <- xb * xl + yb * yl
    f[10]<- (xl - xr)^2 + (yl - yr)^2 - L^2

    deltt <- dy - f
    deltt[9:10] <- -f[9:10]

    list(deltt)
  })
}

eps <- 0.01; M <- 10; k <- M * eps * eps/2
L <- 1; L0 <- 0.5; r <- 0.1; w <- 10; g <- 9.8
yini <- c(xl = 0, yl = L0, xr = L, yr = L0,
          ul = -L0/L, vl = 0, ur = -L0/L, vr = 0,
          lam1 = 0, lam2 = 0)
library(rootSolve)
rootfun <- function (dyi, y, t)
  unlist(caraxis(t, y, dy = c(dyi, 0, 0),
    parms = NULL)) [1:8]
dyini <- multiroot(f = rootfun, start = rep(0,8),
  y = yini, t = 0)$root
(dyini <- c(dyini,0,0))

[1] -0.500000  0.000000 -0.500000  0.000000  0.000000 -9.799999  0.000000 -9.799999
[9]  0.000000  0.000000

caraxis(t = 0, yini, dyini, NULL)

[[1]]
[1] 2.512380e-09 0.000000e+00 2.512380e-09 0.000000e+00 0.000000e+00 8.108556e-07
[7] 0.000000e+00 8.108556e-07 0.000000e+00 0.000000e+00

index <- c(4, 4, 2)
times <- seq(from = 0, to = 3, by = 0.01)

```

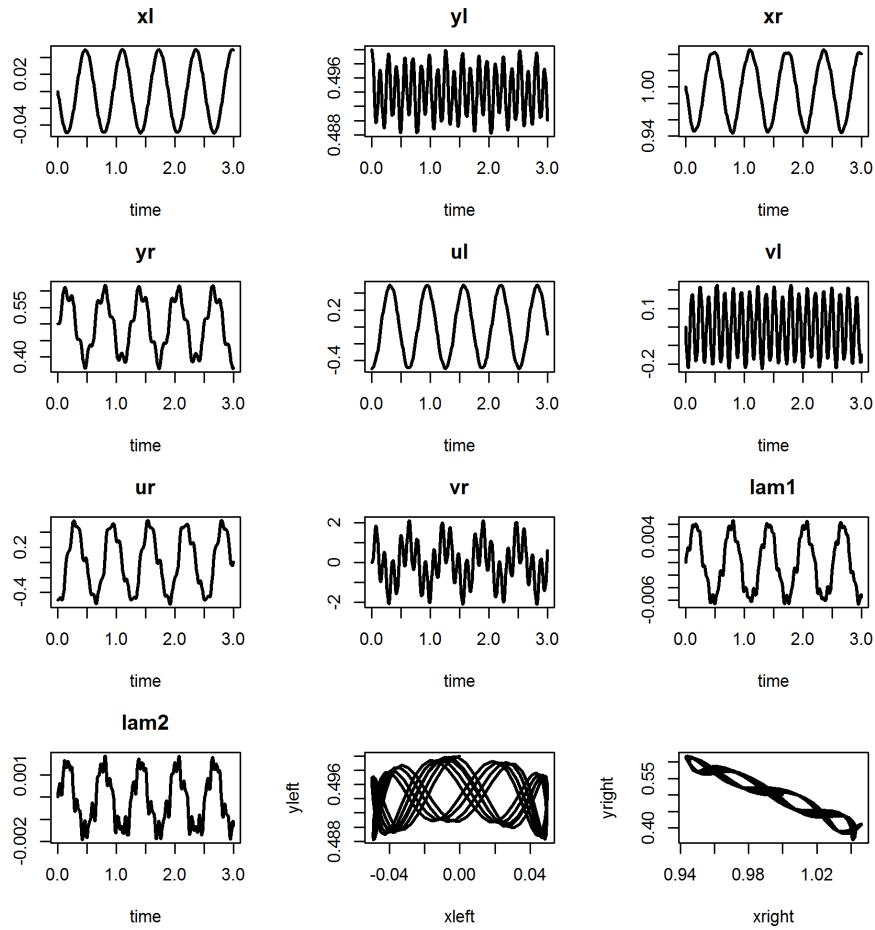


Figure 2: The car axis problem. See book for explanation.

```
out <- mebdfi(y = yini, dy = dyini, times = times,
             res = caraxis, parms = parameter, nind = index)
```

```
par(mar = c(4, 4, 3, 2))
plot(out, lwd = 2, mfrow = c(4,3))
plot(out[,c("xl", "yl")], xlab = "xleft", ylab = "yleft",
     type = "l", lwd = 2)
plot(out[,c("xr", "yr")], xlab = "xright", ylab = "yright",
     type = "l", lwd = 2)
```

5. The Transistor Amplifier

```

library(deSolve)
Transistor <- function(t, u, du, pars) {
  deltax <- vector(length = 8)
  uin <- 0.1 * sin(200 * pi * t)
  g23 <- beta * (exp( (u[2] - u[3]) / uf) - 1)
  g56 <- beta * (exp( (u[5] - u[6]) / uf) - 1)

  deltax[1] <- (u[1] - uin)/R0
  deltax[2] <- u[2]/R1 + (u[2]-ub)/R2 + (1-alpha) * g23
  deltax[3] <- u[3]/R3 - g23
  deltax[4] <- (u[4] - ub) / R4 + alpha * g23
  deltax[5] <- u[5]/R5 + (u[5]-ub)/R6 + (1-alpha) * g56
  deltax[6] <- u[6]/R7 - g56
  deltax[7] <- (u[7] - ub) / R8 + alpha * g56
  deltax[8] <- u[8]/R9
  list(deltax)
}
ub <- 6; uf <- 0.026; alpha <- 0.99; beta <- 1e-6; R0 <- 1000
R1 <- R2 <- R3 <- R4 <- R5 <- R6 <- R7 <- R8 <- R9 <- 9000
C1 <- 1e-6; C2 <- 2e-6; C3 <- 3e-6; C4 <- 4e-6; C5 <- 5e-6
mass <- matrix(nrow = 8, ncol = 8, byrow = TRUE, data = c(
  -C1,C1, 0, 0, 0, 0, 0, 0,
  C1,-C1, 0, 0, 0, 0, 0, 0,
  0, 0,-C2, 0, 0, 0, 0, 0,
  0, 0, 0,-C3, C3, 0, 0, 0,
  0, 0, 0, C3,-C3, 0, 0, 0,
  0, 0, 0, 0, 0,-C4, 0, 0,
  0, 0, 0, 0, 0, 0,-C5, C5,
  0, 0, 0, 0, 0, 0, C5,-C5
))
yini <- c(0, ub/(R2/R1+1), ub/(R2/R1+1),
  ub, ub/(R6/R5+1), ub/(R6/R5+1), ub, 0)
names(yini) <- paste("u", 1:8, sep = "")
ind <- c(8, 0, 0)
times <- seq(from = 0, to = 0.2, by = 0.001)
out <- radau(func = Transistor, y = yini, parms = NULL,
  times = times, mass = mass, nind = ind)

plot(out, lwd = 2, which = c("u1", "u5", "u8"),
  mfrow = c(1, 3))

```

Affiliation:

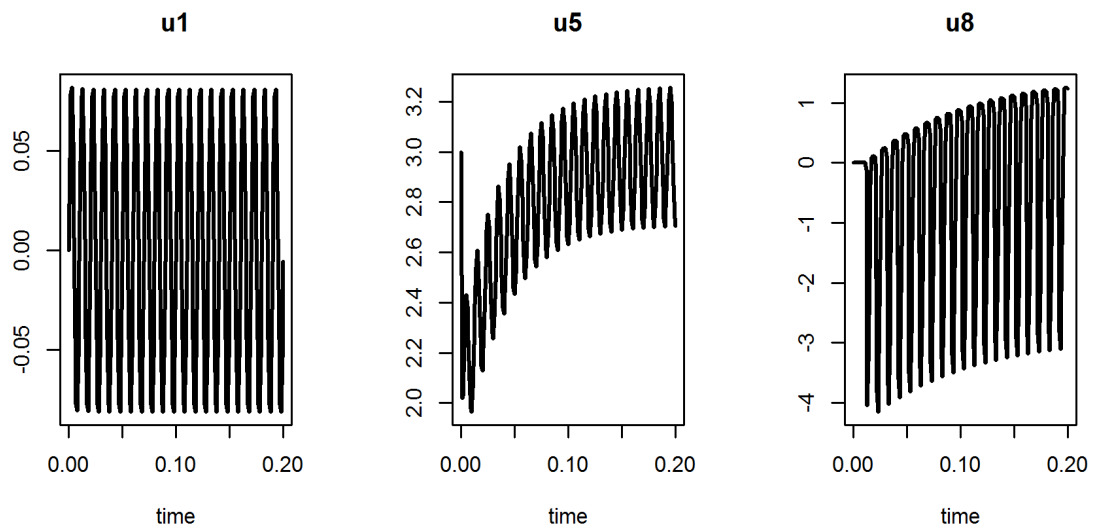


Figure 3: The transistor amplifier. See book for more information.

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