

Solving Differential Equations in R (book) - DDE examples

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Abstract

This vignette contains the R-examples of chapter 8 from the book:
Soetaert, K., Cash, J.R. and Mazzia, F. (2012). Solving Differential Equations in R.
that will be published by Springer.

Chapter 8. Solving Delay Differential Equations in R.

Here the code is given without documentation. Of course, much more information
about each problem can be found in the book.

Keywords: delay differential equations, initial value problems, examples, R.

1. Two simple examples

```
DDE1 <- function(t, y, parms) {  
  tlag <- t - 1  
  if (tlag <= 0)  
    ylag <- 1  
  else  
    ylag <- lagvalue(tlag)  
  
  list(dy = - ylag, ylag = ylag)  
}  
yinit <- 1  
times <- seq(from = 0, to = 10, by = 0.1)  
yout <- dede(y = yinit, times = times, func = DDE1,  
             parms = NULL, atol = 1e-10, rtol = 1e-10 )  
tt <- which(times >= 1 & times <= 2)  
analytic <- c(1-times[times <1] , 0.5*times[tt]^2 - 2*times[tt]+3/2)  
max(abs(yout[times <= 2,2] - analytic))
```

```
[1] 1.388897e-10
```

```
DDE2 <- function(t, y, parms) {  
  tlag <- t - 1
```

```
if (tlag <= 0)
  ylag <- 1
else
  ylag <- lagderiv(tlag)

list(dy = - ylag, ylag = ylag)
}
yout2 <- dede(y = yinit, times = times, func = DDE2,
             parms = NULL )
```

2. Chaotic Production of White Blood Cells

```
mackey <- function(t, y, parms, tau) {  
  tlag <- t - tau  
  if (tlag <= 0)  
    ylag <- 0.5  
  else  
    ylag <- lagvalue(tlag)  
  dy <- 0.2 * ylag * 1/(1+ylag^10) - 0.1 * y  
  list(dy = dy, ylag = ylag)  
}  
yinit <- 0.5  
times <- seq(from = 0, to = 300, by = 0.1)  
yout1 <- dede(y = yinit, times = times, func = mackey,  
             parms = NULL, tau = 10)  
yout2 <- dede(y = yinit, times = times, func = mackey,  
             parms = NULL, tau = 20)  
  
plot(yout1, lwd = 2, main = "tau=10",  
     ylab = "y", mfrow = c(2, 2), which = 1)  
plot(yout1[, -1], type = "l", lwd = 2, xlab = "y")  
plot(yout2, lwd = 2, main = "tau=20",  
     ylab = "y", mfrow = NULL, which = 1)  
plot(yout2[, -1], type = "l", lwd = 2, xlab = "y")
```

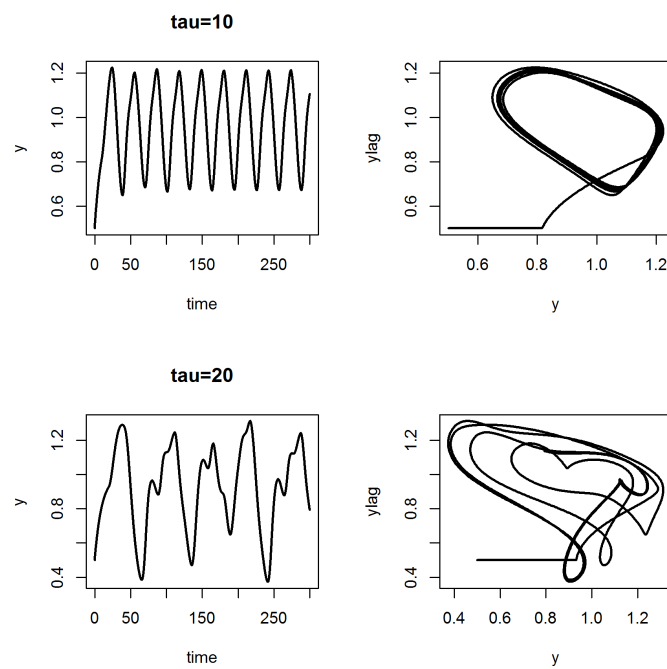


Figure 1: The Mackey-Glass DDE. See book for more information.

3. A DDE involving a Root Function

```

xb <- -0.427; a <- 0.16; xi <- 0.02; u <- 0.5; tau <- 1
yinit <- c(y = 0.6)
mariott <- function(t, y, parms) {
  tlag <- t - 12
  if (tlag <= 0)
    ylag <- 0.6
  else
    ylag <- lagvalue(tlag)

  Delt <- ylag - xb
  sDelt <- sign(Delt)

  dy <- (-y + pi*(a + xi*sDelt - u*(sin(Delt))^2))/tau
  list(dy)
}
times <- seq(from = 0, to = 120, by = 0.5)
yout <- dede(y = yinit, times = times, func = mariott,
            parms = NULL)
root <- function(t, y, parms) {
  tlag <- t - 12
  if (tlag <= 0)
    return (1) # not a root
  else
    return(lagvalue(tlag)- xb)
}
event <- function(t, y, parms) return(y)
yout <- dede(y = yinit, times = times, func = mariott,
            parms = NULL, rootfun = root,
            events = list(func = event, root = TRUE))
attributes(yout)$troot

[1] 14.01588 24.49263 67.54677 75.18143 118.43615

plot(yout, lwd = 2,
     main = "Controller problem")
abline(v = attributes(yout)$troot, col = "grey")

```

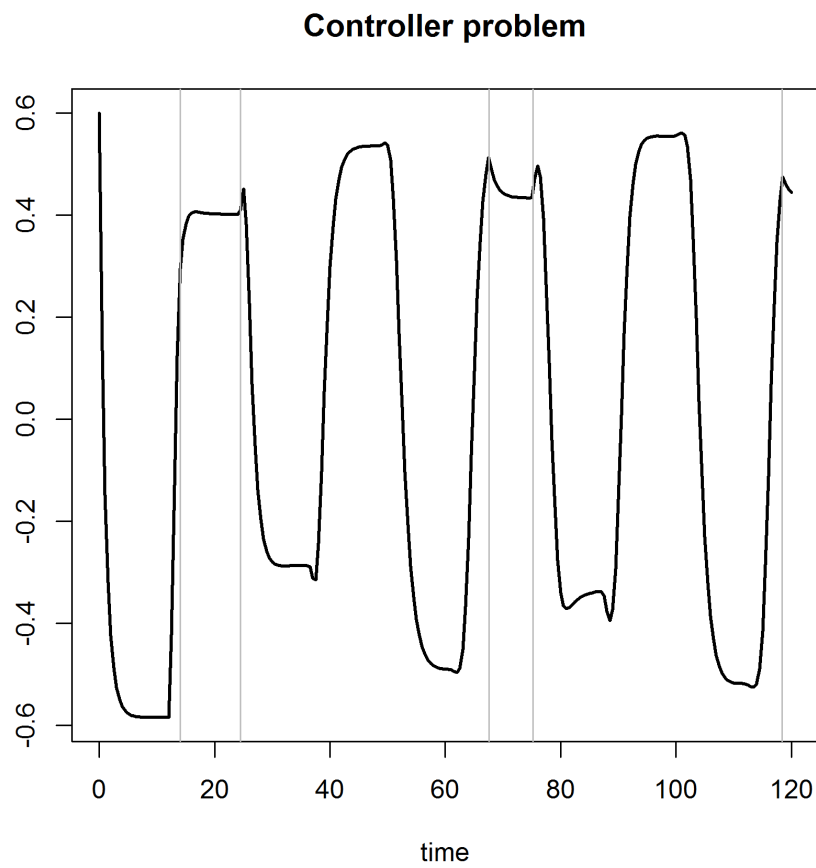


Figure 2: Solution of the Controller problem. See book for explanation.

4. Vanishing Time Delay

```
vanishing <- function(t, y, parms, cc) {  
  
  tlag <- t*y^2  
  if (tlag <= 0) {  
    ylag <- 0  
    dylag <- 0  
  } else {  
    ylag <- lagvalue(tlag)  
    dylag <- lagderiv(tlag)  
  }  
  dy <- cos(t)*(1+ylag) + cc*y*dylag +  
    (1-cc)*sin(t)*cos(t*sin(t)^2) - sin(t+t*sin(t)^2)  
  
  list(dy)  
}  
yinit <- c(y = 0)  
times <- seq(from = 0, to = 2*pi, by = 0.1)  
yout <- dede(y = 0, times = times, func = vanishing,  
  parms = NULL, cc = -0.5,  
  atol = 1e-10, rtol = 1e-10)  
print(max(abs(yout[,2] - sin(yout[,1]))))  
  
[1] 1.81828e-06
```

5. Predator-Prey Dynamics with Harvesting

```

LVdede <- function(t, y, p) {
  if (t > tau1) Lag1 <- lagvalue(t - tau1) else Lag1 <- yini
  if (t > tau2) Lag2 <- lagvalue(t - tau2) else Lag2 <- yini

  dy1 <- r * y[1] *(1 - Lag1[1]/K) - a*y[1]*y[2]
  dy2 <- a * b * Lag2[1]*Lag2[2] - d*y[2]

  list(c(dy1, dy2))
}
rootfun <- function(t, y, p)
  return(y[1] - Ycrit)
eventfun <- function(t, y, p)
  return (c(y[1] * 0.7, y[2]))
r <- 1; K <- 1; a <- 2; b <- 1; d <- 1; Ycrit <- 1.2*d/(a*b)
tau1 <- 0.2; tau2 <- 0.2
yini <- c(y1 = 0.2, y2 = 0.1)
times <- seq(from = 0, to = 200, by = 0.01)
yout <- dede(func = LVdede, y = yini, times = times,
             parms = 0, rootfun = rootfun,
             events = list(func = eventfun, root = TRUE))
attributes(yout)$troot [1:10]

[1]  2.125283  3.057600  3.991063  4.926748  5.864435  6.803803  7.745041  8.688137
[9]  9.632305 10.578815

plot(yout[, -1], type = "l")

```

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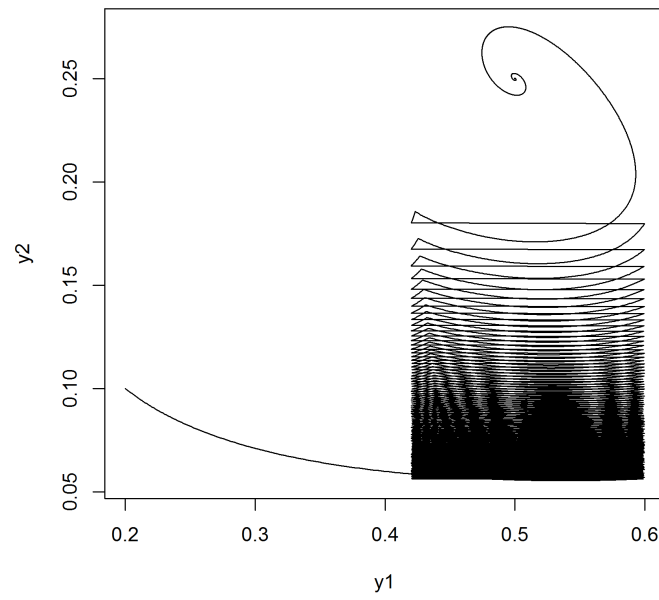


Figure 3: Solution of the predator-prey DDE model. See book for explanation.